Kick-Starting Discussions with the FLIPPED CLASSROOM

A four-phase process and three principles for building a mathematics learning community use rich discussion of student work.

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So begins a typical day in our university precalculus classrooms:

***Susan*:** Jason, do you think that is right?
***Jason*:** No, I think it should be 0 minutes, because everything is 0 at the reference point.
***Malique*:** I'm not sure about that, Jason. It took 18 minutes to get from Topeka to mile marker 363.
***Susan*:** Well, is that the time, or is that the change in time?
***Malique*:** What's the difference?

As students arrive, they settle into their small groups and begin sharing their ideas and questions from the homework assigned the previous night. It can be quite challenging to create a learning environment in which students engage in mathematical thinking outside class and are prepared to share their discoveries and questions with peers the following class period. In this article, we describe how we leverage the flipped classroom model to create such a community.

We believe in the power of discussion and the potential that it holds for students' sense making in mathematics. Sometimes, however, days of planned instruction stretch into weeks because more time is needed for students to express their reasoning and critique the reasoning of others. As teachers transition from lecture-based to discussion-based teaching and learning, they might wonder, "How can I build a mathematics learning community using rich discussion of student work and still have time to keep the intended pace of the course?" The flipped classroom model has helped us find an answer to this important question. However, we are aware that the flipped classroom does not automatically enhance mathematical sense making (NCTM 2014). Here we describe a process to facilitate classroom discussions, present principles for flipped classroom instruction, and use a classroom vignette to illustrate how we implement this process with students in our courses.

FOUR PHASES OF DISCUSSION

Mathematical discussions in our classrooms are supported by flipped instruction and are guided by a four-phase progression that is rooted in the mathematics education literature (Smith, Bill,

and Hughes 2008; Rubenstein, Beckmann, and Thompson 2004). First, we present students with a task and briefly draw their attention to prior mathematical knowledge that they might use to complete the task (the *launch* phase). Second, we provide sufficient time for students to explore and formulate their solutions to the task (the *exploration* phase). Third, we select students to present their work to the class (the *presentation* phase). Fourth, we engage in a class discussion of the presented work (the *discussion* phase). Most of our university students have never experienced a sustained exploratory and discussion-oriented learning environment and require substantial time to orient themselves during the launch and the beginning of the exploration phase. Therefore, we use the flipped classroom to facilitate these first two phases—to kick-start, if you will, the subsequent mathematical discussions.

Here is how this approach works. We design out-of-class instruction to solidify students' mathematical understandings developed in previous classes and prepare them for tasks that they will complete in an upcoming class session. To accomplish this, we create two-to-five-minute videos (using a screencast tablet app such as Knowmia®, ScreenChomp®, or Explain Everything™) that recap related mathematical ideas from previous class sessions, launch students' exploration of a new task, and solicit initial student solutions. Students communicate their initial solutions to us using technology (by either typing their response or taking a picture of their work with a smartphone and sharing through email or Dropbox® service). Students who complete this instructional process before class are prepared for in-depth investigations during the subsequent class period. Moreover, this process allows us to formatively assess students' mathematical thinking before class begins.

FLIPPED-CLASSROOM PRINCIPLES TO DEVELOP SENSE MAKING

As we plan our in-class and out-of-class flipped instruction, we follow the general flipped classroom principles listed below (Strayer, forthcoming). The goal of our flipped instruction is always to develop a sense-making community of mathematics learners. Indeed, these principles can help all teachers heed NCTM's warning in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014) that many flipped classrooms attend more to the use of technology as a vehicle for lecture than the promotion of conceptual learning and student sense making (p. 80). The vignette that we present illustrates how we apply these principles to accelerate our move to the discussion phase of instruction while teaching an inverse functions lesson using the flipped classroom model. We do not necessarily follow these principles sequentially because all three principles inform the decisions we make at different moments during our flipped instruction.

The three flipped classroom principles are these:

- *Principle 1*: Use out-of-class tasks to encourage student reflection and elicit a response from students.
- *Principle 2*: Use in-class tasks to build new knowledge as part of a learning community.
- *Principle 3*: Connect out-of-class and in-class tasks using the same instructional approach.

AN INVERSE FUNCTIONS LESSON

In our university precalculus classrooms, we teach students using the reform-minded and research-based precalculus materials *Pathways to Calculus* (Carlson, Oehrtman, and Moore 2013). The instructional goal for the task in the following vignette was to develop a definition for inverse functions that the entire classroom community—that is, teacher and students—could agree on and use.

Launch

To begin, we created a two-minute homework video (tinyurl.com/MT-inverses) that reminded students of a big idea that we had encountered during a recent class discussion related to function composition—namely, that functions can be composed together so that the output of one function becomes the input of a second function. In keeping with principle 1, the video asked students to reflect on the special case of two functions that "undo" each other when composed together (i.e., the case of inverse functions). The video elicited responses by posing the problem in **figure 1** (along with a few similar problems) and

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asking students to communicate their responses to us the night before the next class.

Explore

We launched this task outside class using the flipped classroom model. As a result, students began to imagine what it might mean for one function to "undo" another function before we developed a working definition of what it means for one function to be the inverse of another. Students' responses to part (ii) revealed the various ways in which they formulated a function h that would undo the multiplication by 5 process of function f (see **fig. 2**). We took advantage of this variety in student responses when planning instruction for the following class session.

Because we value student sense making, we began class by presenting students with a list of their peers' responses from the out-of-class task (see **fig. 2**), challenged them to determine in small groups which of the proposed formulas for h really "undid" the function f, and asked them to defend their choices in a discussion with their peers (per principle 3). Selecting and sequencing the student work from the previous day's homework for in-class discussion induced a conflict for students to resolve.

Coauthor Hart leveraged the induced conflict to achieve the lesson goal of developing a shared definition of function inverses. Initially, his students struggled in small groups to determine and defend correct responses. To help students accept their own mathematical authority in the classroom community, Hart made an in-the-moment instructional decision and challenged students to complete the following sentence as a way of moving forward: "If a function h undoes a function f, then . . . ."

Students attempted to complete this stem in small groups. One group of students proceeded in the following way:

Malique**:** Abby, you have some good-looking answers to last night's homework. How should we finish this sentence?

Abby**:** If a function h undoes a function f, then the output of h is always the input of f.

Malique**:** I don't know about that, Abby. I think it should be "If a function h undoes a function f, then the input of h is always the output of f."

Jason**:** Yeah, I agree, Malique. The output of f must always be the input of h. [*Notice that this statement is reflected in formulations 5 and 8 in* ***fig. 2****.*]

Present

Abby's group then wrote Malique's sentence on the whiteboard, while another group wrote the equivalent of Abby's original sentence on the whiteboard; a third group was stumped. Notice that neither Malique's nor Abby's statement is entirely correct. Not wanting to interrupt student sense making but recognizing the need for additional prompting, Hart drew the picture shown in **figure 3** of a function h that does not undo f yet agrees with the sentences the students had written. He then asked the groups to discuss whether the diagram correctly depicts an inverse function and the statements they had written.

Several students in the "stumped" group had not completed their homework the night before and were encountering these ideas for the first time. Nonetheless, this group moved forward, as evidenced in the following discussion:

A function f multiplies its input values x by 5; that is, $f(x) = 5x$. For example, if you multiplied an input of 7 by 5, your output would be 35.

(i) What process undoes the process of multiplying each input value by 5?
(ii) Define a function h that undoes the process of f.
(iii) If you input 35 into the function h, the inverse process of f that you defined in (ii), what is the output?

Source: Carlson, Oehrtman, and Moore (2013), p. 101

Fig. 1 This question was launched in the homework video.

(1) $h - f(x)$

(2) $h(f(x)) = \dfrac{5}{x}$

(3) $h(f^{-1}(x)) = \dfrac{x}{5}$

(4) $\dfrac{f(x)}{5}$

(5) $h(f(x)) = \dfrac{f(x)}{5}$

(6) $h(x) = \dfrac{x}{5}$

(7) $h^{-1}(f(x))$

(8) $h(y) = \dfrac{y}{5}$ where $y = f(x)$

(9) $h(x) = \dfrac{f(x)}{5}$

Fig. 2 Students submitted a variety of formulas to answer question (ii).

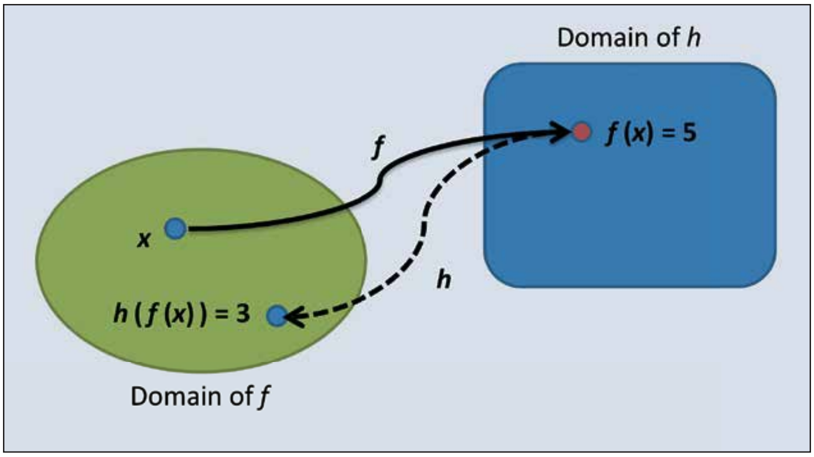

Fig. 3 Coauthor Hart drew this picture to support a class discussion of students' developing understanding of inverse functions.

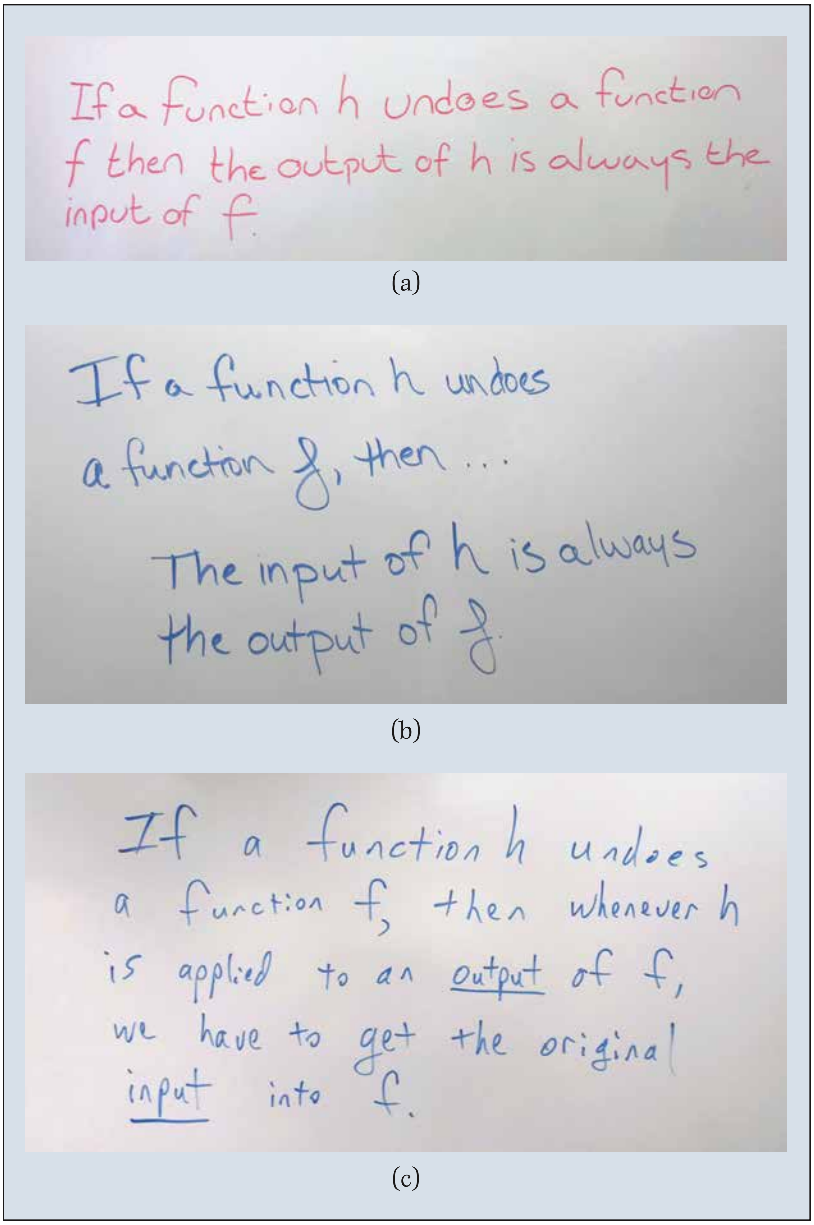

Fig. 4 Students propose a definition: Abby's initial statement (**a**); Malique's revised statement (**b**); Deneae's statement (**c**).

***Deneae*:** Brent, what do you think about the diagram?

***Brent*:** Well, $f(x)$ is defined to be the output of f, so I guess the way Dr. Hart wrote it, the input of h is the output of f.

***Zeke*:** I don't think h really undoes f, though.

***Brent*:** Why not, Zeke?

***Zeke*:** Because h sent $f(x)$ to 3 instead of to x, so h does not undo f.

***Deneae*:** Oh, I think I see it now. Whenever h is applied to an *output* of f, we have to get the original *input* into f.

***Zeke*:** Sweet! Write that up on the board for us.

Discuss

At this point, three groups had written their version of the challenge sentence on the board (see **figs. 4a**, **4b**, and **4c**). Hart called the class together and asked students to compare these three sentences with the diagram in **figure 3**. During the ensuing whole-class discussion, students shared the following thoughts:

***Abby*:** I think the picture on the board does represent my statement. The circle that is the input of f is the circle that h sends all of its outputs to. But . . . I don't know . . . I don't think that is what I *meant* . . .

***Malique*:** Right! The picture represents my statement, too. But . . . it doesn't represent what I was thinking. I think . . . I was trying to say what Deneae said. If h undoes f, then h would send $f(x)$ back to x, and the picture doesn't show that.

***Deneae*:** I agree. If the picture represented my statement, h would need to close the loop and send $f(x)$ back to x.

The class agreed with Deneae. Reasoning from the visual representation of the function composition, members of the class said it was "obvious" that h in the diagram did not undo f because it did not "close the loop" from x to $f(x)$ and back to x. Having eliminated the first two sentences from consideration, the class determined that Deneae's sentence was needed for h to undo f.

With the definition from Deneae's group in hand, Hart sought to connect students' developing understanding of inverse functions to the out-of-class work once again (see principle 3). He returned the small groups to their original planned task—determining which, if any, of the solutions submitted from the homework the night before (see **fig. 2**) really undo the function $f(x) = 5x$. He suggested that they use their newly constructed definition (see **fig. 4c)** and consider the case where $x = 7$ is the input value for f.

Here is an example of the resulting discussion:

Janelle: Formula 1 does not work at all—h should use division, not subtraction.
Willard: Right, Janelle, and formula 2 doesn't work either, because we should divide by 5, not by x.
Jesse: I guess formula 3 is kinda composing h with itself—that doesn't work.
Janelle: Right, Jesse. Formulas 4 and 7 don't tell us what to do when we input something into h, but I don't know about the others. They all look kinda the same.
Willard: Yeah, Janelle. I don't know about the others. I mean, don't formula 6, $h(x) = x/5$, and formula 5, $h(f(x)) = f(x)/5$, just say the same thing?
Janelle: Maybe. But I don't get how x can be one thing and then another in the same problem . . . if formula 6 is correct, then you start with $x = 7$ in f earlier in the problem, but later you say $x = 35$ in h for formula 6. I don't get it.
Jesse: Yeah, Janelle, I get what you're saying.

Janelle's dilemma exposed students' difficulty with understanding x as an input variable that can take on any number of different values. At this point, Hart concluded this task through whole-class discussion.

It may be surprising that, because the student groups' conversations occurred simultaneously, all the discussion took only about thirty minutes.

Dr. Hart: Everybody, remember that when we are working with functions, x is not a fixed value but can take on many different values. It's really just a placeholder.
Brent: Yeah, OK. If that's true, I guess [formula 6] is probably the best answer.
[*A large majority of the class nods in agreement.*]
Susan: I don't know, Brent. Since h needs to work on the output of f, I think [formula 8] is better.
James: Why is that, Susan?
Susan: Well, it's got $f(x)$ as the input into h.

Of course, Susan's observation is correct in some respects. With more mathematical sophistication

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than we would usually say in precalculus, we know that a function f has an inverse if and only if f is a *bijection* onto its range. In this regard, every input y into the inverse h is really $f(x)$ for some x in the domain of f. Hart acknowledged Susan's concern, but, not wanting to enter into a theoretical discussion on the merits of that concern, he closed the exercise by saying that formula 6 represents the inverse in the most general way as a function in its own right.

It may be surprising that, because the student groups' conversations occurred simultaneously, all the discussion in this vignette took only about thirty minutes. Subsequently, the students completed a second problem in which they developed the inverse of the function that converts degrees Celsius to degrees Fahrenheit.

Using the flipped classroom model to elicit student responses before class made it possible for Hart's students to engage in a deep discussion during class while reasoning with multiple representations of functions (symbols, words, and diagrams). It would have been difficult, if not impossible, for students to experience such rich discussion in one thirty-minute class segment had they not engaged in the launch and the explore phases of the first task before class. More important, because Hart had solicited and selected a list of student responses before class, he was able to use the variety of student solutions presented to the class (see **fig. 3**) and the subsequent discussion to motivate the community of learners to take ownership of an abstract topic, something that could not have occurred without the flipped classroom kick-start.

OUTSIDE "THINK TIME" PAVES THE WAY

Student learning in a mathematical community can be facilitated through the launch, exploration, presentation, and discussion of mathematical tasks. In the past, this four-phase process could easily take up an entire class session for just one task. Our use of flipped instruction to kick-start the launch and exploration phases, however, presents one possible way to help students complete multiple in-depth tasks in one class session by "judiciously adopt[ing] technology that supports effective instruction" (NCTM 2014, p. 80).

With this model of flipped instruction, students take the necessary time to think individually about critical mathematical concepts before engaging in community presentation and discussion. This initial "think time" promotes equity among those students who struggle to express their thinking when first introduced to new mathematical ideas during class. Moreover, teachers are able to monitor student work before class so that we are better prepared to facilitate productive mathematical discussions in class. Most important, we see this approach to flipped instruction as one that can help teachers maintain consistency both inside and outside class, so that students acquire improved mathematical practices of sense making while also gaining a deep and connected understanding of a course's key mathematical ideas.

REFERENCES

Carlson, Marilyn P., Michael Oehrtman, and Kevin Moore. 2013. *Pathways to Calculus: A Problem-Solving Approach; Student Workbook*. 4th ed. Phoenix, AZ: Rational Reasoning.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Rubenstein, Rheta N., Charlene E. Beckmann, and Denisse R. Thompson. 2004. *Teaching and Learning Middle Grades Mathematics*. Hoboken, NJ: Wiley.

Smith, Margaret S., Victoria Bill, and Elizabeth K. Hughes. 2008. "Thinking through a Lesson: Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School* 14 (3): 132–38.

Strayer, Jeremy F. Forthcoming. "Designing Instruction for Flipped Classrooms." In *Instructional-Design Theories and Models: The Learner-Centered Paradigm of Education*, edited by Charles M. Reigeluth, Rodney D. Myers, and Brian J. Beatty. New York: Routledge.



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Switch colors
2-in-1
EXPO
DRY ERASE
Just like that.
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EXPO
dry erase markers
NEW 2 IN 1
NEW!